Mesh Representations

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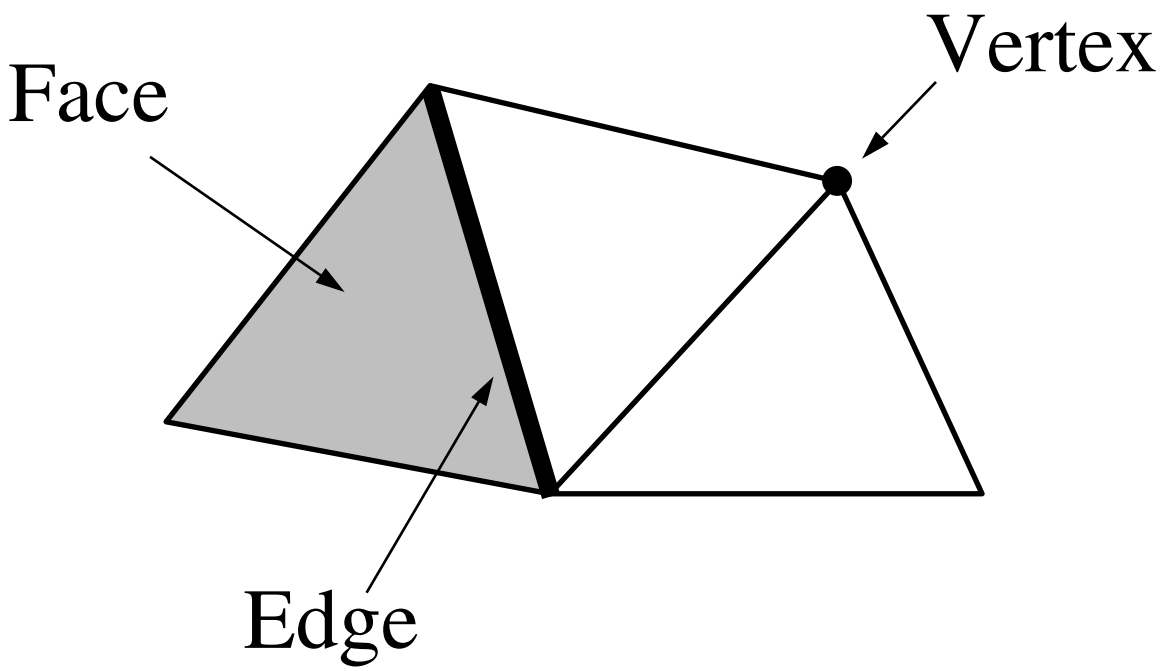
Today's Lecture

- **The Beauty of CS105**
  - Viewing of our 3D model
- **Programming Assignment #2**
  - Scene graph optimization
- **Mesh Operations**
  - Traversal operations
  - Euler operations
  - Compound operations
- **Mesh Representations**
  - Set of Faces
  - Triangle strips
  - Vertex tables
  - Adjacency lists
  - Winged–edge

Mesh Representations

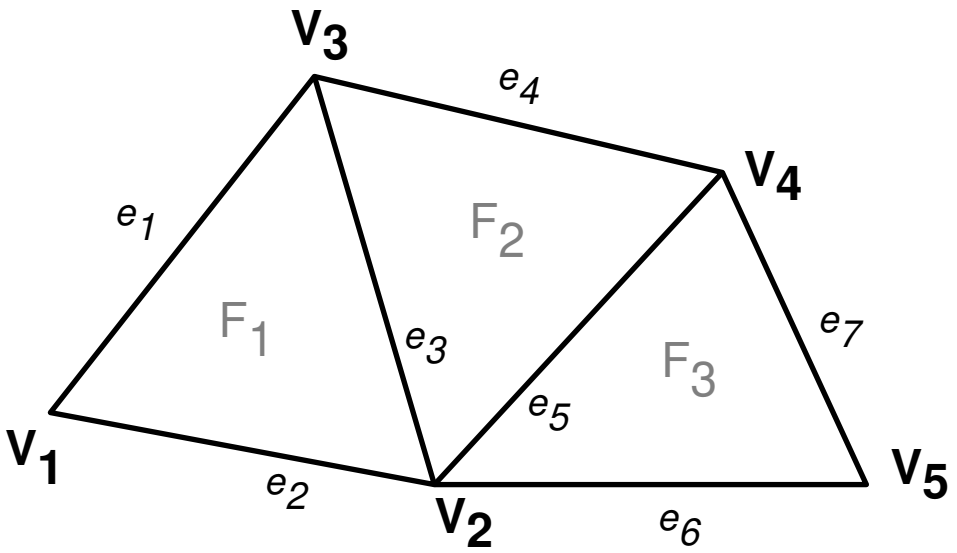
- **Boundary representation**
  - Solids not explicitly represented
- **Piecewise linear**
  - Approximate curved surfaces



Mesh Operations

- **Topological traversal**
  - For each face, enumerate vertices
  - For a face, find adjacent faces
  - For an edge, find adjacent edges
  - For a vertex, find adjacent faces

- **Topological surgery**
  - Insert vertex on edge
  - Insert edge splitting a face



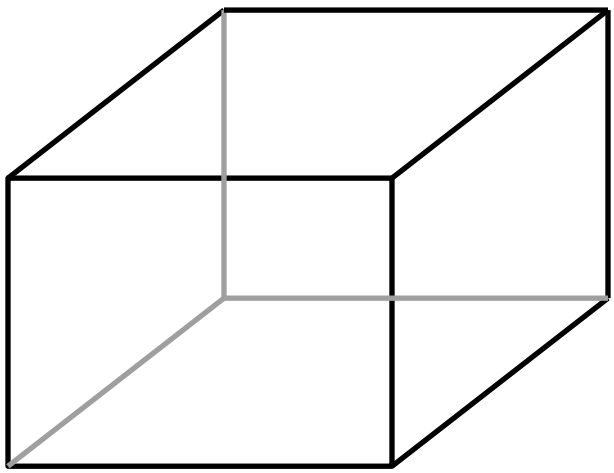
Euler Operations

- **Maintain topological integrity**
  - Insure Euler–Pontcare formula for 3D polyhedra

$$V - E + F - H = 2(M - G)$$

V = # Vertices *E = # Edges*
F = # Faces *H = # Hole loops*
G = # Handles *M = # Objects*

Example:



V = 8
E = 12
F = 6
H = 0
G = 0
M = 1

$$8 - 12 + 6 = 2$$

Euler Operations

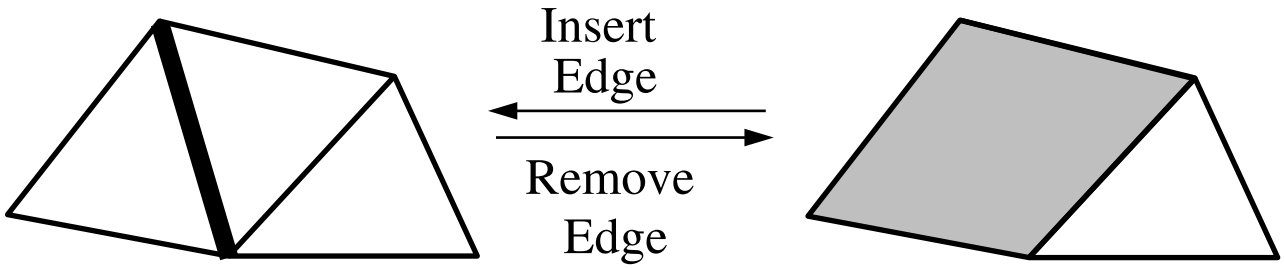
mbfv:	Makes object, face, and vertex
mev:	Creates vertex and edge
splite:	Splits edge into two by inserting vertex
mfe:	Makes face and edge
kbfv:	Removes object, face, and vertex
kev:	Removes vertex and edge
joine:	Joins two edges by removing vertex
kfe:	Removes face and edge
me–kh:	Makes edge, kills hole loop
me–kbf:	Makes edge, kills object and face
mhr–kf:	Makes hole loop and handle, kills face
mh–kbf:	Makes hole loop, kills object and face

$$V - E + F - H = 2(M - G)$$

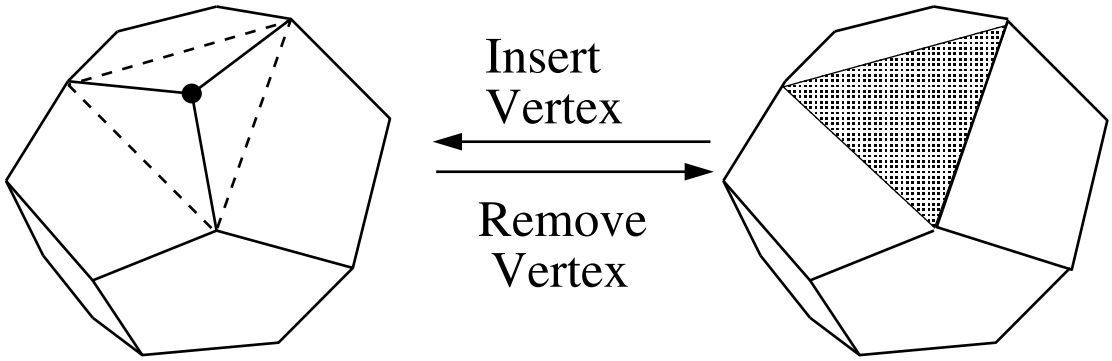
V = # Vertices *E = # Edges*
F = # Faces *H = # Hole loops*
G = # Handles *M = # Objects*

Compound Operations

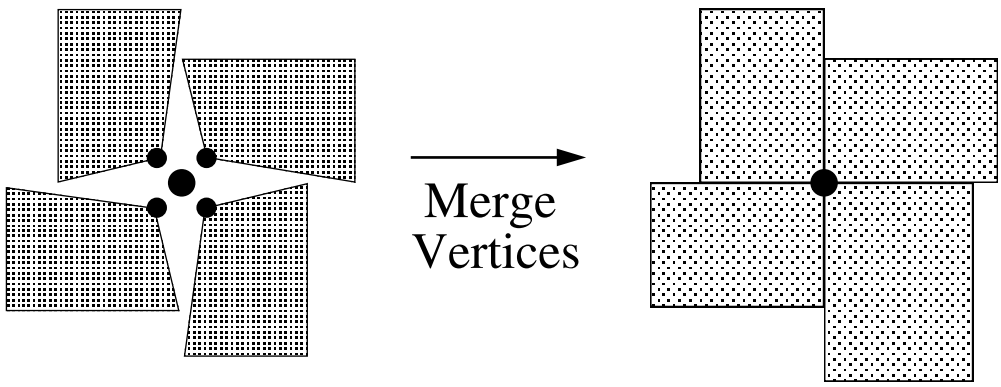
Insert edge: Inserts edge across face
Remove edge: Removes edge while joining two faces



Insert vertex on face: Splits face into many
Remove vertex: Creates face, or joins adjacent faces

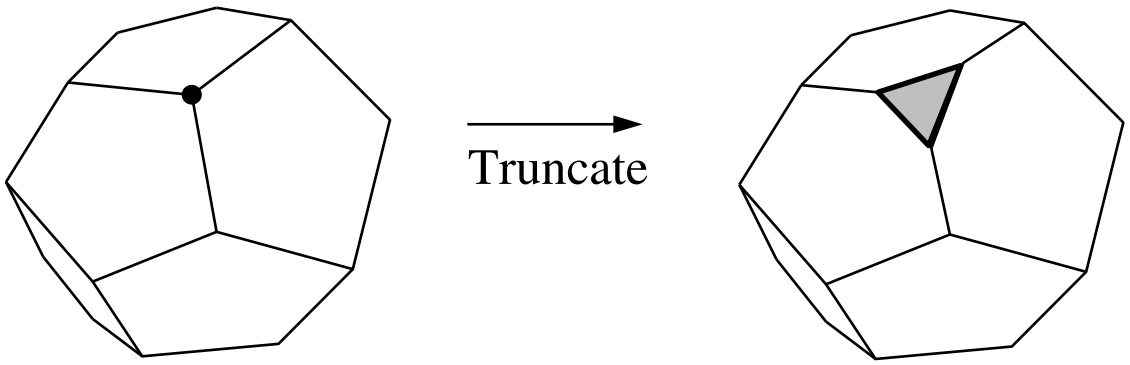


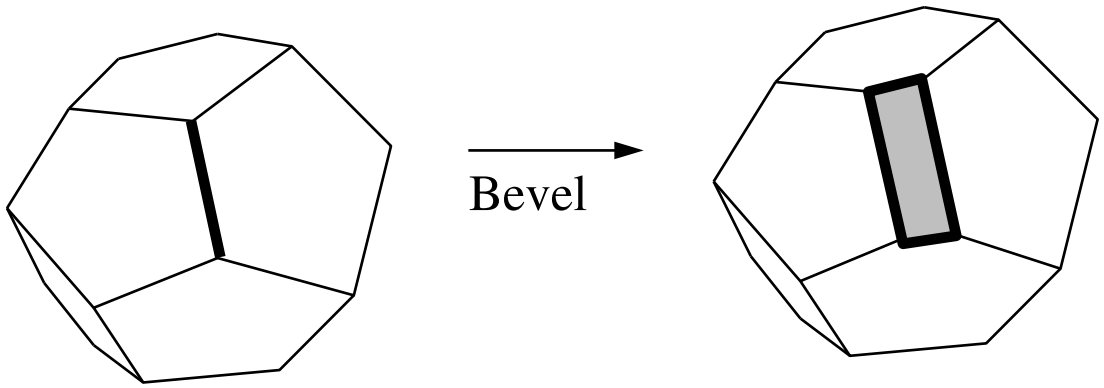
Merge vertices: Collapses n vertices into one



High–Level Operations

Truncate: Replaces vertex by face
Bevel: Replaces edge by face





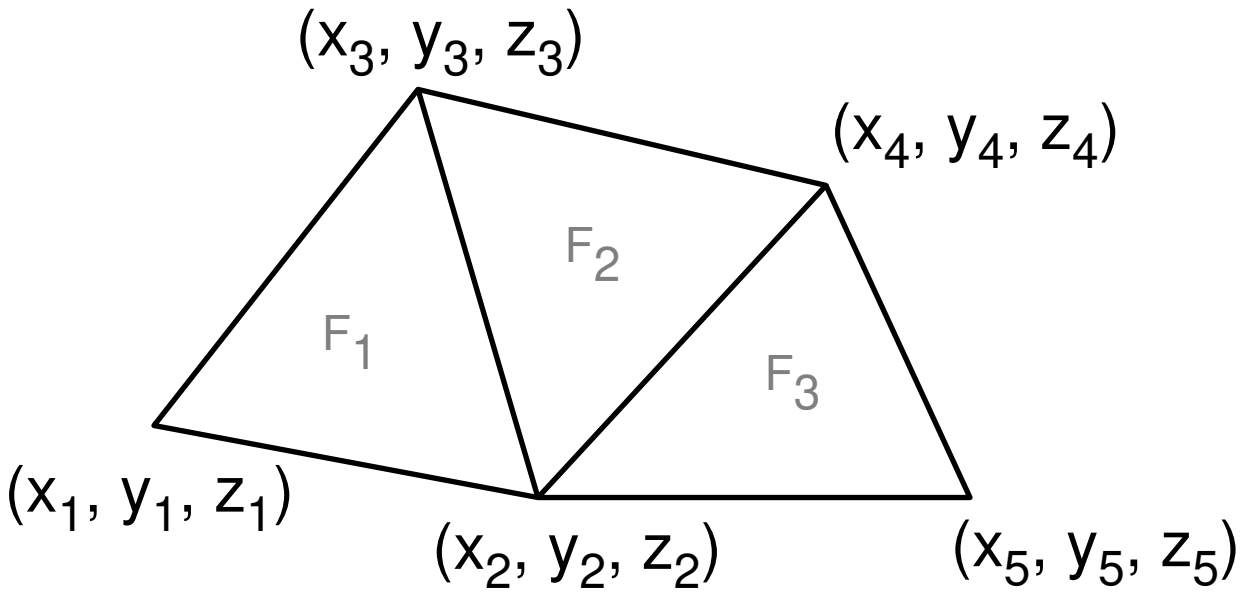
Mesh Representations

- **Representations**
  - Set of Faces
  - Triangle strips
  - Vertex tables
  - Adjacency lists
  - Winged–edge

- **Considerations**
  - Storage requirements
  - Computational efficiency
  - Ease of specification

Set of Faces

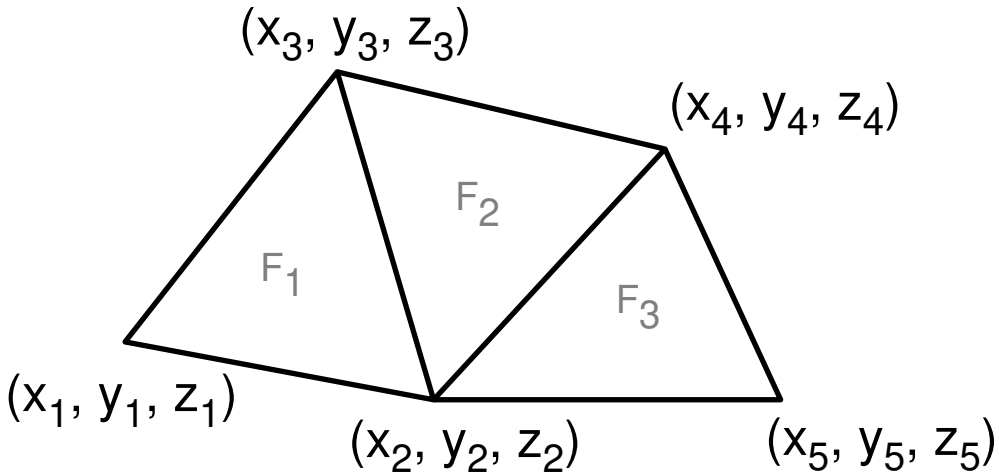
- **Each face lists vertex coordinates ...**
  - Redundant vertices
  - No topology information
  - Not hierarchical or multiresolution



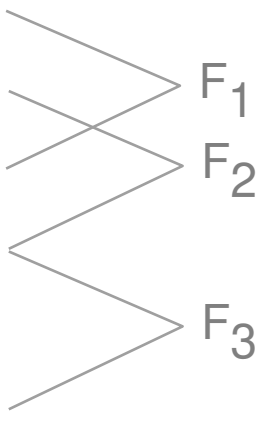
FACE TABLE	
F_1	(x_1, y_1, z_1) (x_2, y_2, z_2) (x_3, y_3, z_3)
F_2	(x_2, y_2, z_2) (x_4, y_4, z_4) (x_3, y_3, z_3)
F_3	(x_2, y_2, z_2) (x_5, y_5, z_5) (x_4, y_4, z_4)

Triangle Strips

- **F_i is triangle (V_i, V_{i+1}, V_{i+2})**
  - Faces are implicit
  - Only k–sided faces
  - Limited vertex sharing
  - Limited adjacency information
  - Not hierarchical or multi–resolution

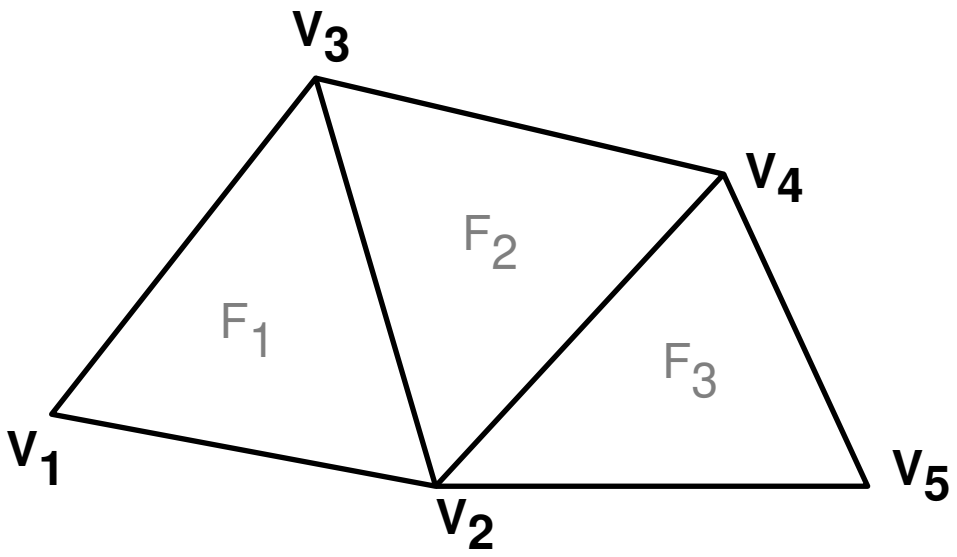


TRI STRIP
(x_1, y_1, z_1)
(x_2, y_2, z_2)
(x_3, y_3, z_3)
(x_4, y_4, z_4)
(x_2, y_2, z_2)
(x_5, y_5, z_5)



Vertex Tables

- **Each face lists vertex references ...**
  - + Shared vertices
  - – No adjacency information
  - – Not hierarchical or multiresolution
  - – Adjacency in O(n) time, generally

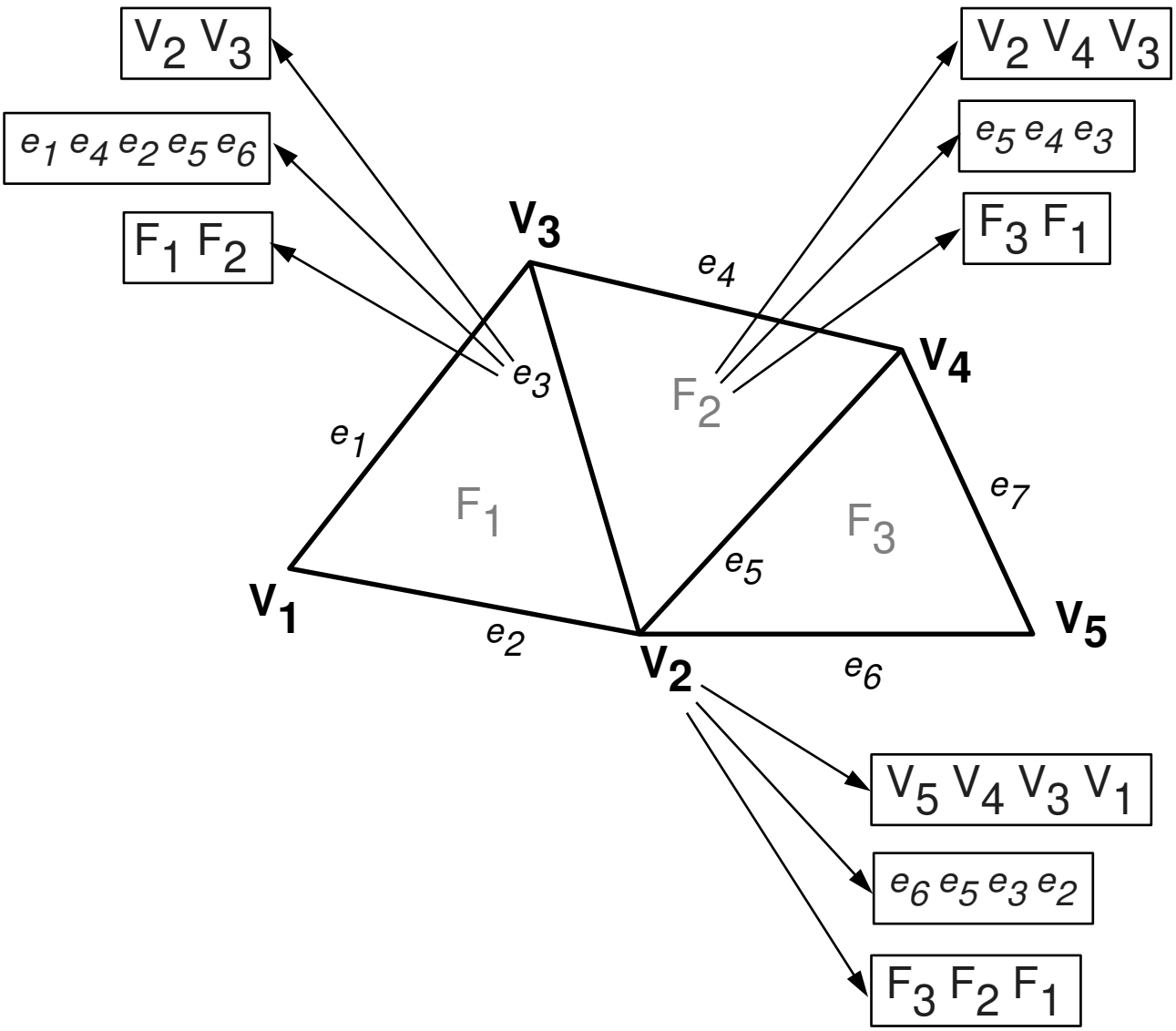


VERTEX TABLE			
V_1	X_1	Y_1	Z_1
V_2	X_2	Y_2	Z_2
V_3	X_3	Y_3	Z_3
V_4	X_4	Y_4	Z_4
V_5	X_5	Y_5	Z_5

FACE TABLE			
F_1	V_1	V_2	V_3
F_2	V_2	V_4	V_3
F_3	V_2	V_5	V_4

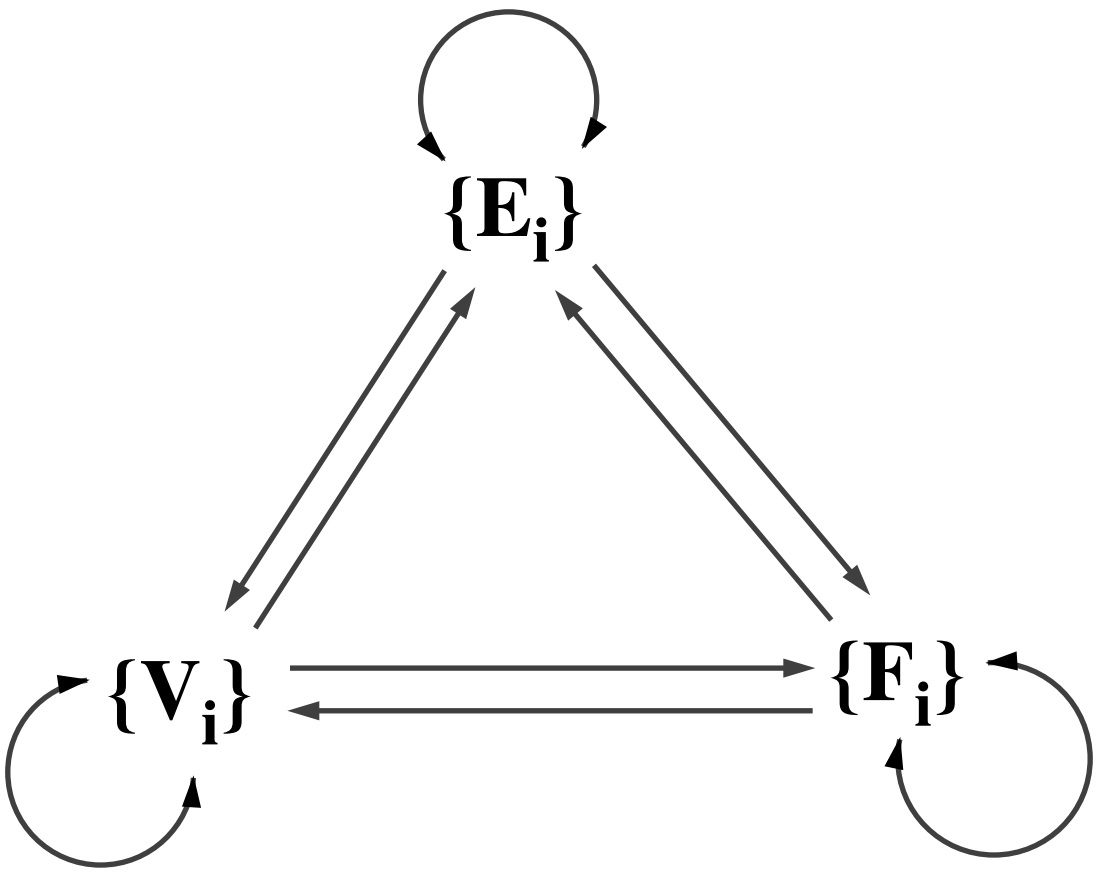
Adjacency Lists

- **Store all adjacency relationships**
  - Adjacency in one lookup
  - Efficient topology traversal
  - Extra storage



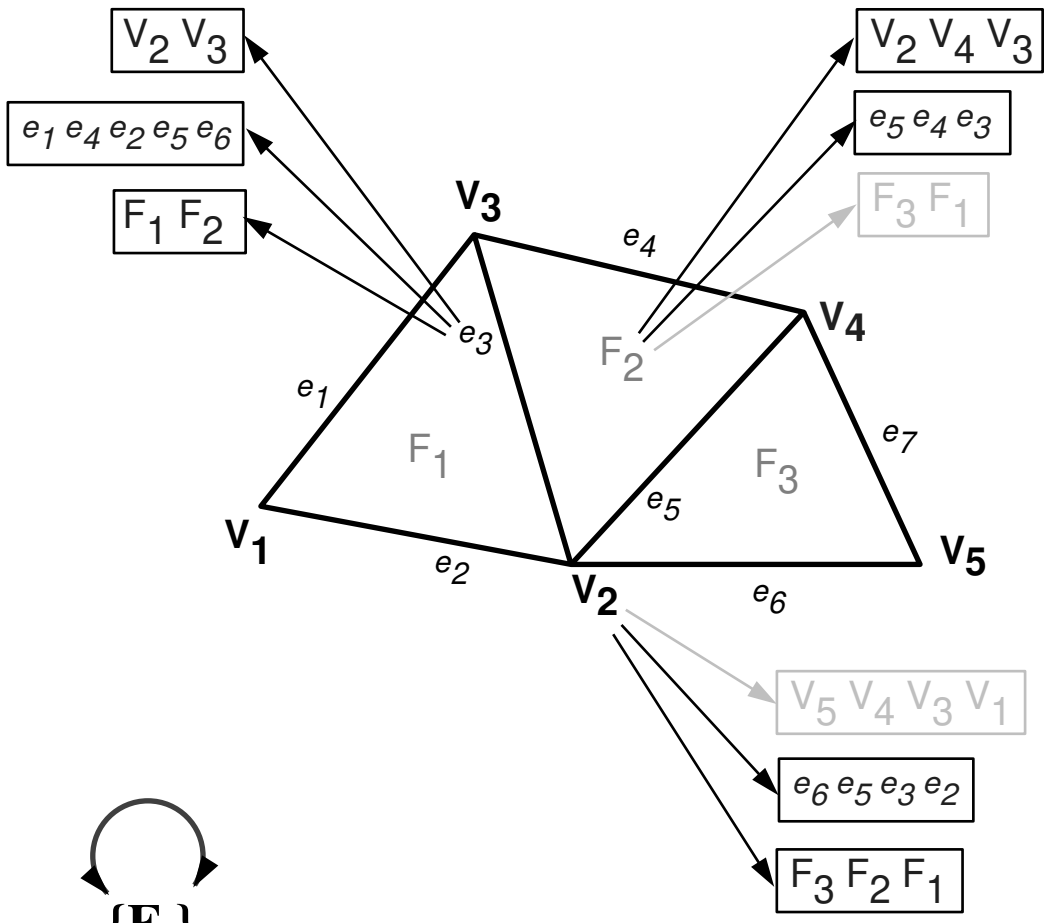
Adjacency Lists

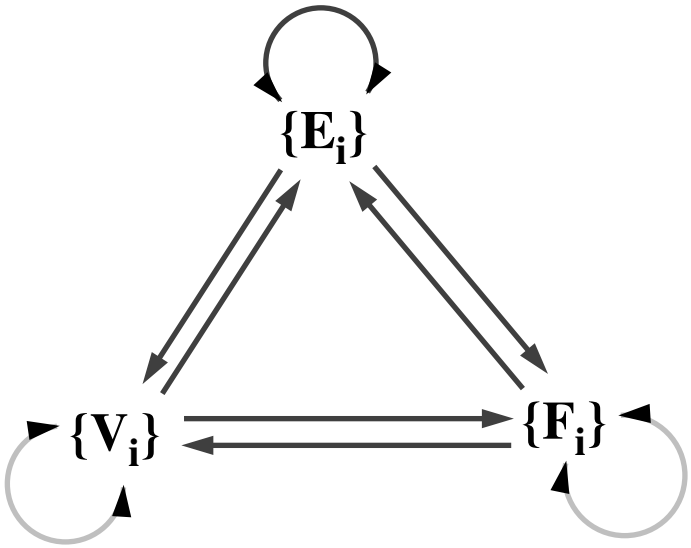
- **Directed Graph Schematic** (Woo '85)
  - Directed edge = explicit relation
  - Directed path = implicit relation



Partial Adjacency Lists

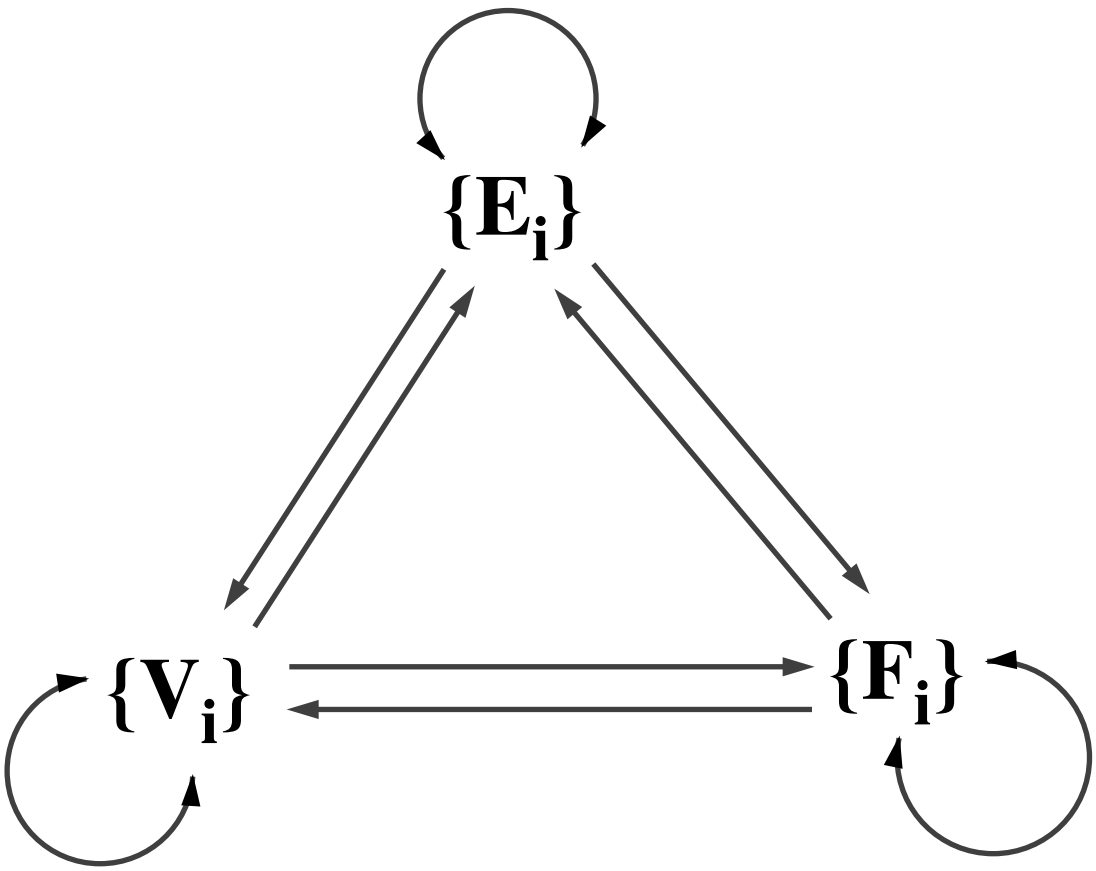
- **Store some adjacency relationships and derive others**





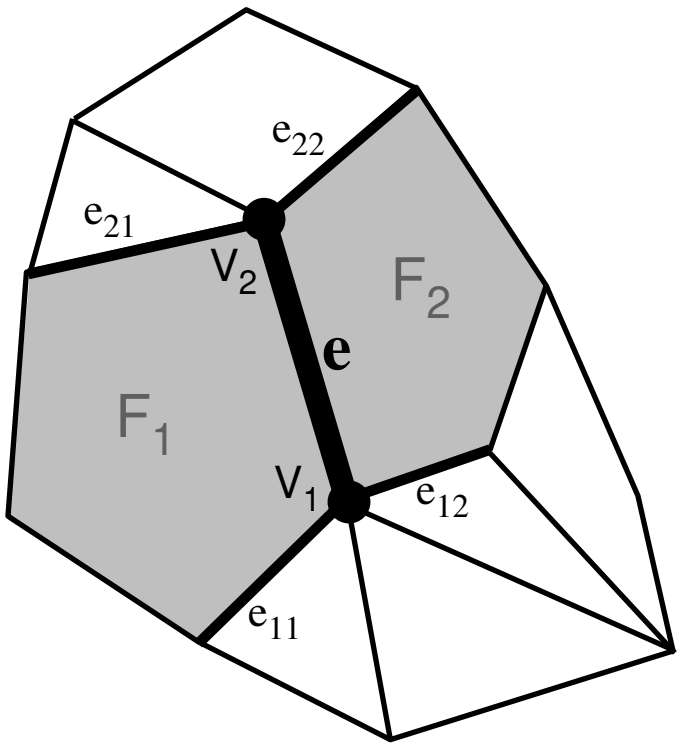
Partial Adjacency Lists

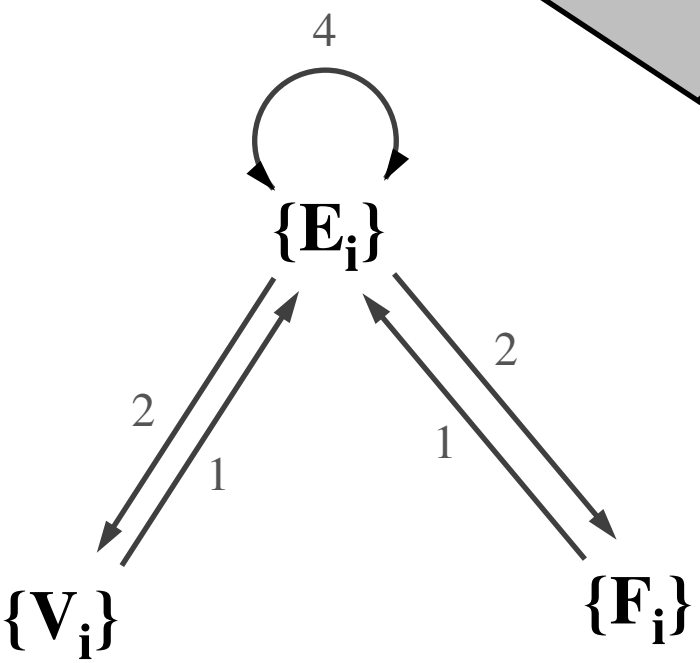
- **Which relations should be stored?**
  - Maintain fast adjacency queries
  - Use least storage



Winged Edge

- **Adjacency encoded in edges** (Baumgart '72)
  - Adjacency in O(1) time
  - Little extra storage (fixed records)





Winged Edge

EDGE TABLE

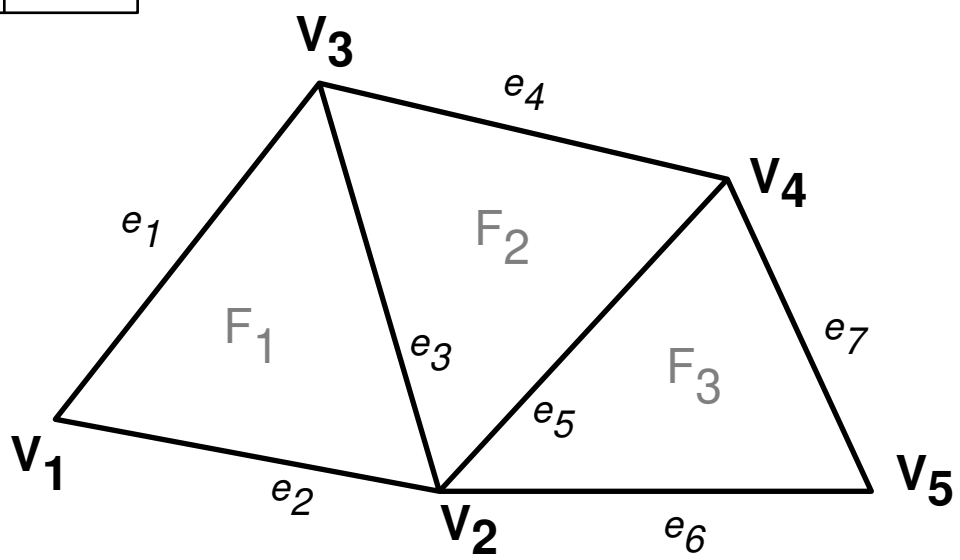
					11	12	21	22
e_1	V_1	V_3		F_1	e_2	e_2	e_4	e_3
e_2	V_1	V_2	F_1		e_1	e_1	e_3	e_6
e_3	V_2	V_3	F_1	F_2	e_2	e_5	e_1	e_4
e_4	V3	V_4		F_2	e_1	e_3	e_7	e_5
e_5	V_2	V_4	F_2	F_3	e_3	e_6	e_4	e_7
e_6	V_2	V_5	F_3		e_5	e_2	e_7	e_7
e_7	V_4	V_5		F_3	e_4	e_5	e_6	e_6

VERTEX TABLE

V_1	X_1	Y_1	Z_1	e_1
V_2	X_2	Y_2	Z_2	e_6
V_3	X_3	Y_3	Z_3	e_3
V_4	X_4	Y_4	Z_4	e_5
V_5	X_5	Y_5	Z_5	e_6

FACE TABLE

F_1	e_1
F_2	e_3
F_3	e_5



Winged Edge

- **Deriving adjacency relationships**

F –> e_1:

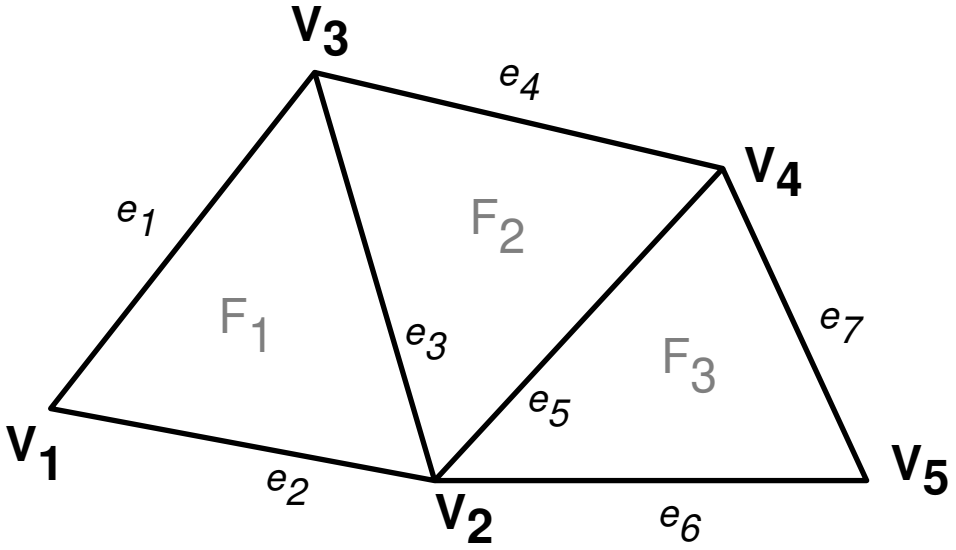
return F.Edge();

F, e_i –> e_{i+1}:

if (e_i.Face(1) == F) return e_i.Edge(2,1);
else return e_i.Edge(1,2);

F, e_i –> V_i:

if (e_i.Face(1) == F) return e_i.Vertex(1);
else return e_i.Vertex(2);



Summary

- **Mesh Operations**
  - Traversal operations
  - Euler operations
  - Compound operations
- **Mesh Representations**
  - Set of Faces
  - Triangle strips
  - Vertex tables
  - Adjacency lists
  - Winged–edge
- **Mesh Considerations**
  - Storage requirements
  - Computational efficiency
  - Ease of specification
- **Programming Assignment #3**
  - Mesh manipulation